Hearing Beats—

For the person who's trying to hear beats for the first time, it's helpful to reduce distracting noises, so strip muting the piano from say F2 to F5 or so might help, or strip the whole piano. Secondly, it's good to have the piano in tune to start with as a starting point that is already a good listening point.

It seems to me that when a person is asking for help in hearing beats, it's the fast beating intervals (major thirds and sixths, and minor thirds) that are hardest. First of all just try listening to major thirds in the octave above the bass break. It seems like some thirds pop out more than others. I'm not sure how much of that is the scaling of the piano and how much relates to a person's ear. But try to find thirds that are more obvious. Then move to thirds that are less obvious and see if you can find the beat there. Sometimes playing 10^{ths} across the bass break helps to bring out the beat. Tenths are just thirds plus an octave. And they seem to be more obvious than just the thirds. The goal is to get your brain past the tone/pitch of an interval to the beats that are in the background. Just be assured that the beats are there, and they aren't really that hard to discern once you've learned to cut past the pitch. A final tip: It can help if you move your head/ear/body. A new position can help make the beat clearer/louder.

Ghosting—I'm not a huge fan of ghosting, but it seems to help some people focus and it helps to teach the harmonic series and the interval relationships. Ghosting depends on finding the coincidental partial for an interval. Let's take the F3-A3 third. To find the coincident partial where the beat will take place, you need to know that the ratio of a major third is 5:4. This means that the 5th partial of the lower note and the 4th partial of the upper note is the note where the beat occurs. The sequence of partials for F3, then, would be F3 (1st partial), F4 (2nd partial), C5 (3rd partial) F5 (4th partial), and A5 (5th partial). The sequence of partials for A3 would be A3 (1st partial), A4 (2nd partial), E5 (3rd partial) and A5 (4th partial). The 5:4 third, F3/A3 will beat at A5. Now that that is determined, silently hold down F3 and A3; then strike A5 with a sharp staccato blow. Because A5 is the coincidental partial for F3 and A3, the A5 partial which is in the harmonic series of both F3 and A3 will be activated so that the beats will be isolated and the pitches of the notes will be less prominent. Try this going up to F#3/A#3, finding the ghosting note, and listening. Go up and down the middle of the piano to find ghosted thirds that may be louder and easier to hear.

Tune—The intervals we tune can all be tuned pure and in fact aural tuning relies on knowing where pure is. It's impossible to properly temper an interval if you don't know where the note should be in regard to pure. The second bit of knowledge you need to have is the character of each interval in regard to pure. Thirds are significantly widened from pure. Fifths are slightly narrow from pure. Fourths are slightly widened from pure. Don't think of a note as being sharp or flat; think of it as being positioned wide or narrow.

Tune a pure major third—You are working on a piano that is already in tune, so you can be assured that the thirds are already wide. Therefore, ask yourself what do I have to do to make a pure third. Let's take G3/B3. Every interval has two choices. Lower/raise the lower string or lower/raise the upper string of the interval. To narrow the third you can either raise G3 or lower B3. Let's lower B3 (taking a string down is safer in regard to breaking a string). You can take some comfort in the fact that your ear naturally wants no beats. A third without beats is calm to the eardrum. But tuning a pure interval is a real test of your motor skills and your auditory skill. You have to control the string movement very finely in order to hear the beat rate slow down and stop. It's easy to get it wrong and move through pure to the narrow side. You're trying to find pure and to do that you need to know where it is. If you go narrow, the third will really start to sound unnatural. That should help you know you've gone too far. Plus if the third is narrow, it will start to beat again. Raise and lower the string until there are no beats. Having a mentor to guide you can be very helpful in this exercise.

Listen—Listen to some other intervals that have B3. Listen to the 4th, B3/E4. It will be beating fairly fast because as you narrowed the third, G3/B3, you've widened the 4th, B3/E4. Any interval that you choose that contains the B3 will be different, i.e., out of tune. Listen to all the intervals and think about how each interval was changed by moving the B3 down. What happened to the B3/D# third? What happened to the E3/B3 fifth? The F#3/B3 4th? The interrelationship of intervals is what makes it possible to aurally tune an equal temperament with accuracy. Like a rubics cube, making all the sides line up is tricky.

Restore the third—Now you need to restore the third by widening it from pure. How do you know if it is correct? The G3/B3 needs to fit between the third below it and the third above. The important thing to listen for is the tempo of the beat. The tempo of G3/B3 is slightly faster than Gflat3/Bflat3 and slightly slower than Aflat3/C4. Use your ETD to check yourself. If you can restore the third within a tolerance of less than 1 cent, you're headed the right direction.

Now do the same exercise only listen to the B3/E4 4th. What do you have to do to make that 4th pure? Fourths are wide. Therefore you have to raise the B3 to make the 4th pure. But the 4th is not tempered to the degree of a third. A 4th is only slightly wide, 1 beat per second, to be exact. The problem with 4ths (and 5ths) is that it is easy to be on the wrong side of pure. As you move the B3 up, you'll probably pop right through pure before you even know it and be on the narrow side. If you hear the beats getting fast as you raise B3, then you know you've slipped past pure. You're on the narrow side and the beats will start speeding up again. If you aren't sure where you are, move the string and listen to what happens. The critical skill for tuning intervals is not counting beats, but hearing the tempo of the beat and hearing that tempo speed up or slow down or stop (pure). As the tempo changes, you're getting information about the character of the interval. You need to know the goal for the interval and then let the tempo of the beat tell you whether you're reaching your goal and whether you've set the string on the correct side of pure for the interval.

An interesting web site that might be helpful:
www.rollingball.com/beats/beat_rates4.html